Future Conventional Oil Production:

Sensitivity to Reserves Uncertainty

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Abstract

This report examines the geotechnical hazard of the peak of conventional oil. The background to the problem is built up and the terms required to understand the study are explained. Literature on how reserves affect production profiles is reviewed. Problems with the publically available sources of oil industry data are highlighted and further reasons for uncertainty – particularly in the six OPEC countries with the largest reserves along with the interesting example of Mexico – are examined. A recently published model is adapted to show how the date of peak changes according to different reserve estimates and is expressed mathematically for the first time. It is applied to the five most important OPEC Middle Eastern countries (Iran, Iraq, Kuwait, Saudi Arabia and UAE) to produce predictions of the date of peak for each from the most pessimistic to the most optimistic of reserve estimates given reasonable assumptions made about the rate of supply growth before, and decline rate after, the peak. Taken together, the study finds that the five countries could have a theoretical group peak date between 2016 and 2037. Finally, the model and its results are critiqued and the possible policy considerations and implications are outlined with the conclusion drawn that the symptoms of peak oil are probably already appearing.

Purpose

A report submitted in partial fulfilment of the requirements for the award of postgraduate certificate in Natural Hazards for Insurers at University College London

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1. Introduction

1.1. Context

1.1.1. Peak oil

Oil wells follow a production curve whereby the observed production rate peaks and then enters a (usually) terminal decline as a result of falling reservoir pressure and/or the breakthrough of water. Because each individual oil well is drilled into a larger fixed oil field, the same effect can usually be observed over an oil field by combining the production rates of individual wells. Aggregating the individual wells shows why almost all oil fields follow this pattern where the production rate grows until the field is completely drilled out and then declines until the field is depleted. Most giant oil fields typically enter into production decline by the time 40% of their resources have been produced (Höök et al. 2009II).

Assuming that oil fields in a basin are discovered (roughly) sequentially at a constant rate, with the largest – which contain most of the oil (IEA 2008) – discovered and exploited relatively early in the exploration history of a region (because they occupy a larger surface area and are more attractive economic targets) until they have all been found, the same effect can again be observed when combining oil fields in a particular region. For example, on a country by country basis, taking the United States, oil production has almost exactly followed the theoretical Hubbert (1956) curve model which first proposed such a production curve. Hubbert's model suggested that large regions tend to enter decline when 50% of their resources have been produced, and

Bentley (2002) shows graphically why individual field production curves, which as per previous paragraph generally enter into decline well before half of their resources have been produced, can combine under the assumptions given to produce this result.

By extension, if one accepts that oil is a geologically limited resource i.e. that there are a finite number of economic oil fields in the world, the same idea can be used again for all the fields in the world. Then, the peak (or under some models, plateau) of world oil production is known as peak oil. It can be defined as the point in time when the maximum rate of global oil production is reached.

1.1.2. A problem of the rate of oil supply

It is important to note that the problem is not one of oil running out as the main concern is not when oil resources will be completely depleted. No-one disputes that current reserves represent over 40 years of production at current rates (BP 2011). Rather, Sorrell et al (2009) identify that it is the rate of production of those resources. For as time goes on, all current fields must enter decline and new oil resources frequently become more expensive to locate, extract, transport and/or refine, and their exploitation often has more severe environmental consequences or requires more energy consumption at all stages of the processing chain. This means that the rate of production of the remaining resources could be relatively lower than the rates that have been experienced so far.

1.1.3. Viewpoints on the peak

The peak oil debate is heavily polarised between optimistic and pessimistic viewpoints (Sorrell et al 2009).

	Optimistic	Pessimistic
Peak	More than two decades away	Already reached or imminent
Factors determining supply	Investment and new technology	Geological
Alternative and non-conventional sources	Developed	Unable to fill gap on timescale required
Economic results	Rising oil prices only	Substantial dislocation

Table 1.1: Optimistic and pessimistic viewpoints on peak oil

Such differences make it politically difficult for the governments of oil importing states to set energy and oil policy for the future (Miller 2011).

1.1.4. Conventional oil

This report analyses sensitivity to reserves uncertainty. Much of the uncertainty, as will be seen later on, comes from Middle Eastern OPEC states as they are the least transparent in their reporting and they have the biggest reserves. The vast majority of the reserves of these Middle Eastern OPEC states are held in the form of conventional oil. Therefore this study concentrates on conventional oil and the questions of whether alternative and non-conventional sources can fill the gap are left beyond the scope of what will be examined.

Different authors treat the definition of conventional oil differently. Conventional oil is generally accepted to include crude oil (light and medium density) and condensate.

However, Campbell (e.g. Campbell and Heapes 2008) excludes oil found in deepwater and in Polar regions, although this is rarely followed by other authors. There are more fervent debates as to whether to include heavy oil (extra heavy oil is generally excluded) and also whether to include Natural Gas Liquids (NGLs) in the definition of conventional oil (McGlade 2010; Sorrell and Speirs 2009). A reasonable definition might be oil that is used for transport fuels which are the primary use of conventional oil. Therefore it would have to be of a viscosity (heavier oils being more viscous) that would naturally flow down a pipeline and also maybe exclude NGLs because they don't contribute much to transport.

Ultimately this study will be constrained by the data used, with each data set using a different definition. This study will, however, provide the definition used by each data source analysed.

1.2. Calculation of Recovery Factor

1.2.1. Oil Originally In Place (OOIP)

OOIP is the total hydrocarbon content in an oil reservoir. It therefore includes both producible and non-producible oil and is estimated using a combination of seismic surveys, measurements of reservoir properties and computer simulation.

1.2.2. Ultimately Recoverable Resource (URR)

Due to reservoir characteristics and the limitations in oil extraction technologies, only a fraction of OOIP can be brought to the surface and produced. This producible part is known as the URR. It is defined by Sorrell et al (2009) as the amount of oil that is estimated to be economically extractable from a field or region over all time (from when production begins to when it finally ends). The concept of URR is challenged by some economists who argue that what will be economically extractable in the future cannot be predicted and these ideas are more thoroughly covered in the Previous Work section of this report. Some authors, for example BGR (2010), use the term Estimated Ultimate Recovery (EUR) for URR.

BP (2011) breaks URR down into constituent parts. These can be displayed in a similar graphical format used by Sorrell et al (2009) as follows:

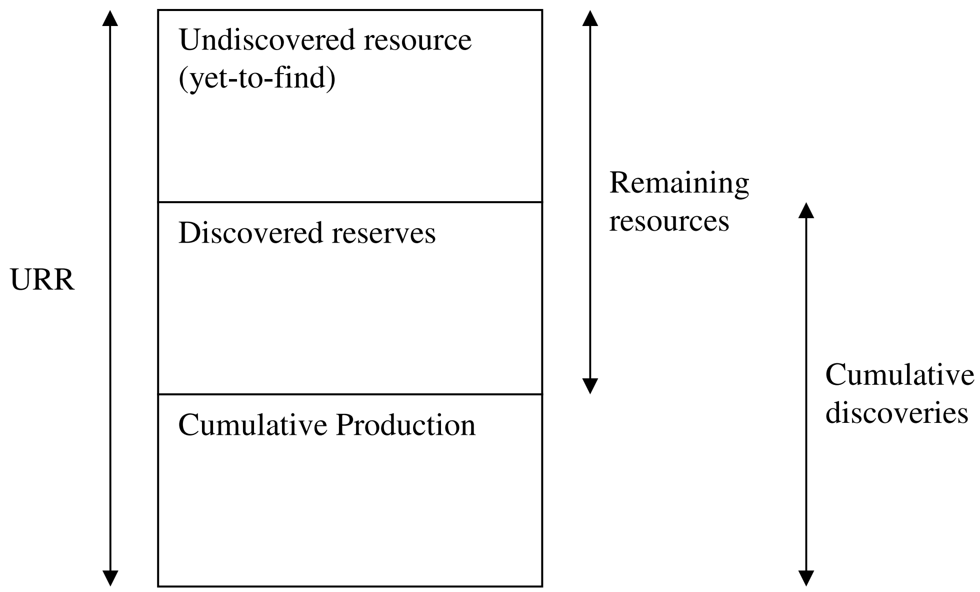

Fig 1.1: Components of URR

1.2.3. Recovery factor

This is defined as the percentage of original oil in place that can be recovered with current or anticipated technology (Sorrell et al 2009) over all time.

It can be expressed mathematically as follows:

$$\text{Recovery Factor } (\%) = \frac{\text{URR}}{\text{OOIP}} \times 100$$

The average global recovery factor is reported by IEA (2008) to be 34.5%, but for particular fields the range of values can be very wide from as high as 80% for high permeability reservoirs (Sorrell et al 2009) down to <10%, depending on the technologies used, the oil quality and the geological structure.

1.3. Depletion and Decline Rates

1.3.1. Reserves depletion rate

Sorrell et al (2009) define this as the rate at which the remaining recoverable resources of a field or region are being produced.

On an annualised basis it gives the proportion of a field's reserves which are produced every year i.e. mathematically:

$$\text{Depletion rate} = \frac{\text{Annual production}}{\text{Estimate of recoverable resources}}$$

Where the Estimate of recoverable resources could be URR or some estimate of remaining reserves

1.3.2. Production decline rate

This is defined as the rate at which oil production from a well, field or region declines (Sorrell et al 2009) over time.

On an annualised basis it yields the change in produced volume from one year to another i.e. mathematically:

$$\text{Decline rate}_{\text{year n}} = \frac{\text{Production}_{\text{year n}} - \text{Production}_{\text{year n-1}}}{\text{Production}_{\text{year n-1}}}$$

1.3.3. Equivalence

Höök et al (2009II) uncovered a strong correlation between depletion rate and decline rate. Further Höök (2009) shows mathematically that for the idealized scenario of production declining at an exponential rate, the depletion rate equals the decline rate. While this assumption of an exponential decline rate is not always a good fit to the data for individual fields as it can often underestimate production during the later stages of a field's life, it generally works well for groups of fields (Miller et al 2009). This is very useful as it is possible to measure production rates directly and country level figures are widely reported so, given this assumption, it is possible to derive reserves depletion rate and therefore also an estimate of recoverable resources on a country-by-country basis from data available in the public domain.

1.4. Reserves

1.4.1. Classification

While the terms used in oil reserves classification are similar and widely used, their meanings vary between entities and jurisdictions due to differences in reporting standards. Essentially the systems used split reserves into three categories which can be thought of as equivalent but not identical.

Type of estimate	Deterministic terminology	Probabilistic terminology	Statistical description
Low	1P: Proved	P90	10th percentile
Best	2P: Proved + Probable	P50	Median
High	3P: Proved + Probable + Possible	P10	90th percentile

Table 1.2: Terminologies associated with oil reserves estimation (source: Sorrell et al (2009))

Taking account of uncertainties in geology, technological possibility and economic viability, 'low', 'best' and 'high' are estimates of recoverable reserves traditionally used internally by companies to assess projects (Sorrell et al 2009). This explains why the middle line is often equated to URR (Bentley et al 2007) before production starts as it is the best estimate of recoverable reserves.

The deterministic terms can have different definitions. Perhaps the most commonly used are those of the Society of Petroleum Engineers (SPE) who define:

<u>Proved Reserves</u> as 'those quantities … which, by analysis of geological and engineering data, can be estimated with reasonable certainty to be commercially recoverable … under current economic conditions, operating methods, and government regulations.'

<u>Probable Reserves</u> as those quantities, prevented by uncertainties of being classified as proved reserves, 'which analysis of geological and engineering data suggests are more likely than not to be recoverable.'

<u>Possible Reserves</u> as those quantities, prevented by uncertainties of being classified as proved reserves, 'which analysis of geological and engineering data suggests are less likely to be recoverable than probable reserves.'

These can be added as outlined in table 4.2 to produce 1P, 2P and 3P reserves figures.

The problem with deterministic terminology is that even the same definition can be interpreted in different ways. Particularly the terms fail to quantify a degree of uncertainty. Probabilistic terms can therefore be used to provide more comparable data. This data is derived by assigning a probability distribution to each of the uncertain factors in a particular project and combining them within a Monte Carlo simulation to give P90, P50 and P10 reserves figures. The P90/P50/P10 figures then give the quantity of oil to have a greater than 90%/50%/10% probability of being recovered.

1.4.2. Aggregation

Due to the more comparable data that is produced by probabilistic terminology, there is a general movement towards their use, with even the conservative US Securities and Exchange Commission (2010) now ruling them to be acceptable. However, using probabilistic terminology poses a problem when aggregating figures. This is because arithmetic addition is generally used to aggregate reserves across wells into fields, companies, countries, regions and the globe.

Particularly Pike (2008) describes that arithmetically adding the P90, which is the most widely available reserve figure in the public domain, of individual assets leads to a figure that significant understates the true P90 of the assets under consideration when taken together. This is because it assumes "that the outcome, simultaneously, of every [asset] will be its 'downside', with no scope for 'upside' beyond the P90 threshold." Correspondingly, it overstates the true P10. With each addition increasing the degree of underestimation/overestimation for aggregated P90/P10 figures, the result is that global estimates are likely to be most biased (Sorrell et al 2009).

The same conclusions may not apply with P50. This is because if the P50 figure is equal to the mean, i.e. if it comes from a symmetrical distribution, there is an equal likelihood of estimating too high or too low so that over a large population of assets the errors should cancel out (Strahan 2007).

However, to aggregate probabilistic estimates of reserves correctly probabilistic addition should be used using Monte Carlo computational techniques (Pike 2006), but unfortunately this is rarely used in practice by authorities and regulators (although oil companies are reported to use it frequently for individual fields).

2. Previous Work

2.1. Peak oil models

Perhaps the model of peak oil that has had the biggest impact so far has been IEA (2008). Governments place a great deal of trust in the IEA using their publications as guidelines for developing future energy policy and this was their first World Energy Outlook report to give a comprehensive analysis of future global oil production tacitly acknowledging the concept of peak oil. There were, however, problems with the Reference Scenario given in the report and these were critiqued by Aleklett et al (2009) and Miller (2011).

Sorrell et al (2009) provides the most comprehensive review of all models of oil peaking and McGlade (2010) builds on that work.

2.2. Reserves and Production profiles

Reserves clearly have a big impact on the date of the peak as they estimate the amount of remaining oil. If production rates remain the same, increasing reserves will delay the date of the peak. There are two possible ways of increasing global reserves, namely raising the recovery factor by using better technology and increasing the sum global OOIP by making new discoveries.

2.2.1. Making new discoveries

This report has already explained why within a basin, fields are typically discovered in approximately descending order of size with the largest of the cheaply accessible fields found near the beginning of exploration. Technologies such as seismic data surveys and visualisation suites are supposed to allow a greater number of smaller fields to be discovered. However, Strahan (2008) quotes Jim Henry former president of the Permian Basin Petroleum Association who says that when the huge reservoirs, which were found first, start declining, the difference can't be made up by the smaller fields found later, because the rates at which they were producing are so huge. Indeed, Sorrell et al (2009) note that around 500 oil fields account for two thirds of all oil discovered so far. They go on to conclude that most of these fields are relatively old and many are past their peak of production and most of the rest will begin to decline within the next decade or so.

While finding any more cheaply accessible fields which are large enough to replace declining production seems unlikely, the argument given by economists goes that as oil prices rise and technology becomes cheaper previously inaccessible or uneconomic fields will be exploited (Strahan 2008). These might, for example, include large fields discovered in deepwater offshore from which oil is technically difficult and therefore expensive to extract. Additionally, the same argument may also cover unconventional sources of oil but these are beyond the scope of this study which is of conventional crude. However, there is no evidence of this argument playing out in reality with Strahan (2008) showing how price does not historically seem to have had any impact on discovery volumes.

New fields being discovered are therefore usually either smaller or technically more difficult to access. Though technical improvements and/or higher prices should make more of them viable, there will always be a lower limit imposed by the energy return on investment (Sorrell et al 2009). This is the theory that the amount of energy required to extract the oil, must be less than the amount of energy that the extracted oil contains, for if it was greater the exercise in extraction would not be worthwhile.

2.2.2. Raising the Recovery factor

With the geological limitation resulting in a fixed worldwide amount for all OOIP and considering that there are now likely to be few new discoveries of any impactful size made, the only way to increase global reserves is to extract and produce more of that OOIP i.e. by raising the recovery factor. As all other inputs are fixed this can only be done by using better technology. Current innovations include activities such as infill drilling, well work-overs, secondary recovery programmes and enhanced oil recovery techniques (IEA 2008), but again these activities are expensive and will produce less affordable oil. It might be tempting to assume that these technologies will undoubtedly delay the date of the peak, but that may not, in fact, be the case.

An analysis by IEA (2008) comparing individual field profiles by vintage showed that fields developed in recent years tend to have shorter and accelerated production profiles with a higher and shorter plateau that is built up to more quickly (and would result in a higher post-peak decline rate) than those developed before the 1990s. They explain this by saying that advances in production technology and (for those fields operated by private companies) pressure from investors to minimise the payback period, have made it financially attractive to introduce improved and enhanced recovery techniques earlier in a field's life.

However, Höök et al (2009I) argue a different production profile saying that technology allows the plateau of giant oil fields to be extended, thereby increasing the proportion of remaining recoverable resources prior to peak, but at the cost of a more rapid decline following the peak.

Either way, both studies reach the same conclusion, that better technology has not been capable of extracting vastly more oil from the oil fields studied. It has a meaningful impact only on the shape of a production curve (e.g. rate of growth, length of plateau, rate of decline, etc.) but not the area underneath it (amount of reserves). Particularly both reach the same conclusion as Simmons (2005) that better technology too often quickly extracts the targeted oil leading to steeper decline rates.

Further, evidence shows that, for a particular field, using better technology usually only reduces the rate of post-peak decline and only exceptionally actually reverses the decline. In fact, the exceptions are so rare that any examples, such as the Weyburn field in Canada (IEA 2008), will be published.

The other argument that might be made for the possibility of increasing recovery factors is evidence of historical changes. As referred to previously, IEA (2008) give an estimate of the global average recovery factor to be 34.5%. However, when comparing this 2008 figure to the earlier published figures of Miller (1995) – who gives 20% -- and Laherrère (2006) – who gives an average calculated by number from

IHS Energy information as 36% from the fields in their 1997 database and 27% from the fields in their 2006 database – no particular trend can be established.

2.2.3. Limitations on reserves increases

In summary, four factors limiting crude oil reserves increases have been explained. These are geology, technology, price and energy return on investment. The sum of these limitations suggests that it will not be possible for reserves to continue to be upwardly revised for ever and therefore that the threat of the peak is undeniable.

3. Data

3.1. Datasets

3.1.1. Possible sources

There are various agencies which report reserves data and a summary of those which are publically available is as follows:

Agency	Res-erves class	Sources	Publicly available references
Oil and Gas Journal (OGJ)	1P	Survey forms	OGJ final issue each year
World Oil	1P	Survey forms	World Oil Magazine August or September issue each year
Organization of Petroleum Exporting Countries (OPEC)	1P	Many sources including member's National Oil Companies, direct communications to the Secretariat, Secretariat's estimates, OGJ, World Oil, BP and OAPEC	Annual Statistical Bulletin
BP	1P	Combination of primary, third party (particularly OPEC, OGJ and World Oil) and independent Assessments	Annual Statistical Review of World Energy
Organization of Arab Petroleum Exporting Countries (OAPEC)	1P	OAPEC Databank, OGJ and OPEC	Annual Report
World Energy Council (WEC)	1P	WEC Member Committees, OGJ, OAPEC, OPEC, World Oil, BP and various national sources	Triennial Survey of Energy Resources
German Federal Institute for Geosciences and Natural Resources	2P	Many sources including Government departments and agencies, OGJ, World	Energy Resources Annual Report

(BGR)		Oil, OPEC, BP, WEC	
IHS Energy (formerly Petroconsultants)	2P	Field-level data	USGS World Petroleum Assessment 2000, IEA (2005), Strahan (2007)
Campbell	2P	Originally produced using Petroconsultants data	Campbell and Heaps (2008), Strahan (2007), www.peakoil.net
Energy Watch Group (EWG)	2P	IHS Energy, authors estimates based on various sources and own assessments	Schindler and Zittel (2008)

Table 3.1: Reserves data sources (compiled from: Bentley et al (2007), Sorrell et al (2009), Thompson et al (2009), McGlade (2010) and references listed)

3.1.2. Countries examined

OPEC is widely regarded as the swing producer in the oil market (i.e. they control a major portion of global reserves and they posses a large spare production capacity allowing them to increase or decrease the supply of oil to the market at minimal additional internal cost thus enabling them to influence prices and balance the market) and their reporting is considered by analysts to be particularly opaque and possibly grossly exaggerated at times. This suggests that their reserve figures will form an interesting basis for this study. The six OPEC countries with the highest reported reserves will be focused on as one would assume that they will have the greatest impact on the peak. These are in descending order of reserves Saudi Arabia, Venezuela, Iran, Iraq, Kuwait and UAE (essentially coming from the emirate of Abu Dhabi). The reserves of Mexico, which is not an OPEC member, but has an interesting history in its reserves reporting in that it had to own up to overstatement, will also be considered.

3.1.3. Historical Reserves

As part of their Annual Statistical Review of World Energy, BP provides a spreadsheet giving reserves figures historically going back to 1980 for all countries. The latest review was published recently in June 2011. The BP data is sourced from or forms the basis of all other 1P data. This, along with the free access and electronic format of the data allowing easy manipulation promoted its use as the initial starting point for analysis of the countries chosen. The data was then plotted in a line chart so that a visual analysis could be performed as follows:

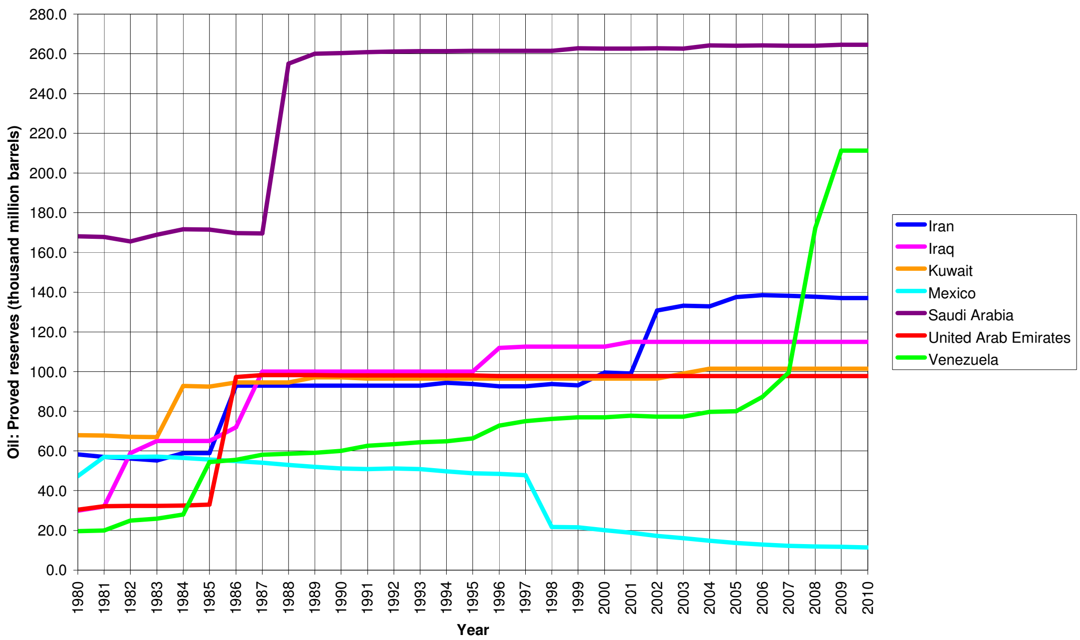

Fig 6.1: Proved oil reserves 1980-2010 (plotted from: BP (2011))

3.2. Analysis of historical proved oil reserves

3.2.1. Flat reserves reporting by OPEC members excluding Saudi Arabia

For the OPEC countries analysed (except Saudi Arabia), reserves are often flat, see: Iran 1986-1993 and 2009-2010; Iraq 1983-1985, 1987-1995, 1997-2000 and 2001-2010; Kuwait 1986-1988, 1991-2002 and 2004-2010; UAE 1987-1995 and 1996-2010; Venezuela 2009-2010.

However, it is recognised that:

$$\text{Reserves}_{\text{year n}} = \text{Reserves}_{\text{year n-1}} - \text{Production}_{\text{year n}} + \text{Discoveries}_{\text{year n}} + \text{Revisions}_{\text{year n}}$$

It seems implausible, therefore, that reserves could possibly remain flat, with no changes at all, over an extended period of years, as it would have to mean that production was exactly equal to new discoveries and revisions (technical improvements) in every year.

Bentley et al (2007) put this exact repetition down to the BP data being sourced from the OGJ and World Oil Magazine who indicated that static data was generated in their figures when the countries in question either had not replied to the survey forms (in which case the journals published the prior years data) or that the countries in question had returned forms identical to the prior year.

Neither explanation gives much confidence in the current reported reserves figures, particularly where they have not been revised recently as in the case of Iraq, Kuwait and UAE. There have been few discoveries of any size in the recent past and, as has been outlined in this report's review of previous work, technological improvements have not improved recovery factors by much, if at all. This leads to the assumption that reserves are being overstated as discoveries plus revisions over recent years are unlikely to have been greater than production.

3.2.2. Reserves jumps by OPEC members in the 1980s

There were big jumps in OPEC members' reported reserves figures in the 1980s. Robelius (2005) argues that while no great discoveries were reported during the years of revisions, some of the increases were justified because the newly nationalised oil companies inherited estimates of reserves from foreign companies that were too low. Indeed, Strahan (2007) judges that the jump in reserves of Saudi Arabia – but notably and specifically not those of any of the other countries – was due to no thorough assessment of reserves having been made prior to nationalisation, explaining that it would be expensive to carry out and would also be unnecessary because known reserves comfortably exceeded the needs of production. However, without any new discoveries, these explanations seem unlikely to be the primary cause of the jumps, particularly when viewed in context of the political environment at OPEC.

What is clear is that during the 1980s there were debates going on within OPEC as to how production quotas should be allocated between each member. At the time there was a glut of oil caused by falling demand following the 1970s energy crises which resulted in a collapse in the oil price which caused economic uncertainty in oil exporting countries. When viewed within the OPEC quota system it becomes apparent that any member would have sound political and economic reasons for wanting to capture as much share of the total quota as they possibly could.

Of particular importance to this study was the use, or perhaps proposed use, of reserves figures in setting quotas. Strahan (2007) says that OPEC discussed a proposal to change the criteria by which quotas were allocated to include the size of each countries reserves but that the new rule was never actually introduced, and in private correspondence he confirmed that his source could be relied on. Whereas Aleklett and Campbell (2003) say that production quotas were based on reserves, Robelius (2005) says that the size of the reserves was a parameter in the calculation of a country's export quota and Bentley et al (2007) say that quotas were driven in part by the size of a country's reported proved reserves. Turning to the organisation itself, OPEC (2009) reports that quotas are agreed by the Conference, although it is unclear whether this is the total production quota for all members or if it is related to how it is shared out between members. A letter was written to the OPEC Secretariat Public Relations and Information Department to clarify the position – a copy of which can be seen in the appendix – but no reply was received and this only further highlights the lack of transparency within the organisation.

Campbell (Aleklett and Campbell 2003; Bentley et al 2007; Strahan 2007, Campbell and Heapes 2008) provides the most comprehensive theory on the reasons behind the jumps on a country by country basis as follows:

- Kuwait increases reported reserves to include cumulative production so that they now give total oil discovered.

- Venezuela increases its reserves by including large amounts of long-known heavy oil that had not been previously reported.
- Iran, Iraq and UAE increase their reserves to be roughly the same as Kuwait as a result of quota competition.
- Saudi Arabia then increases reserves to maintain its quota by doing the same as Kuwait and including cumulative production.

3.2.3. Overstatement of reserves by Mexico

An interesting case in the context of this study is that of Mexico which owned up to overstating reserves. In 1991 Spanish language weekly news magazine Proceso published a report called 'The lies of Pemex', Pemex being the national oil company of Mexico. Reuters (1991) outlines the report, which was based on an interview with Francisco Inguanzo, a deputy director of Pemex, who claimed that Mexico had lied about its oil reserves repeatedly since 1977 in a bid to shore up its financial position. Indeed this turned out to be true as Mexico restated its reserves in 1997, cutting them by 30%. Reports from the time suggest that the overstatement was to raise the international prestige of Mexico as well as the value of its collateral which was used for a borrowing binge that eventually led to the financial collapse of the country in the early 1980s (Krauss 1997). It is interesting that Mexico was able to get away with this overstatement of its reserves for many years without any significant challenges, and this suggests that it would be possible for other countries to be doing the same.

3.2.4 Late 2000s rise in reserves of Venezuela

This was an uprating of reserves after certification of additional proven reserves in the country's Orinoco region (Walter 2007). It relates to extra heavy crude oil which puts it outside the scope of this study as it would have to be synthetically processed to turn it into conventional crude. Interestingly, the reported intention of their oil minister, Raphael Ramirez, at the time was to demand a big increase in its production allowance once OPEC formally recognised the uprating (Milner 2006). This again suggests that reserves do play a role in OPEC quota allocations.

3.3. Reasonable reserve ranges

While being interesting case studies, Mexico and Venezuela will be excluded from further analysis in this report, the former because it is no longer a major international reserve holder after its revisions, and the later because its reserves are in the form of extra heavy oil which is outside the scope of this study into conventional oil.

Therefore the reserves of each of the Middle Eastern countries selected will be examined more closely. 1P data in the BP Annual Statistical Review already examined will be compared with 2P data from the variously publically available sources.

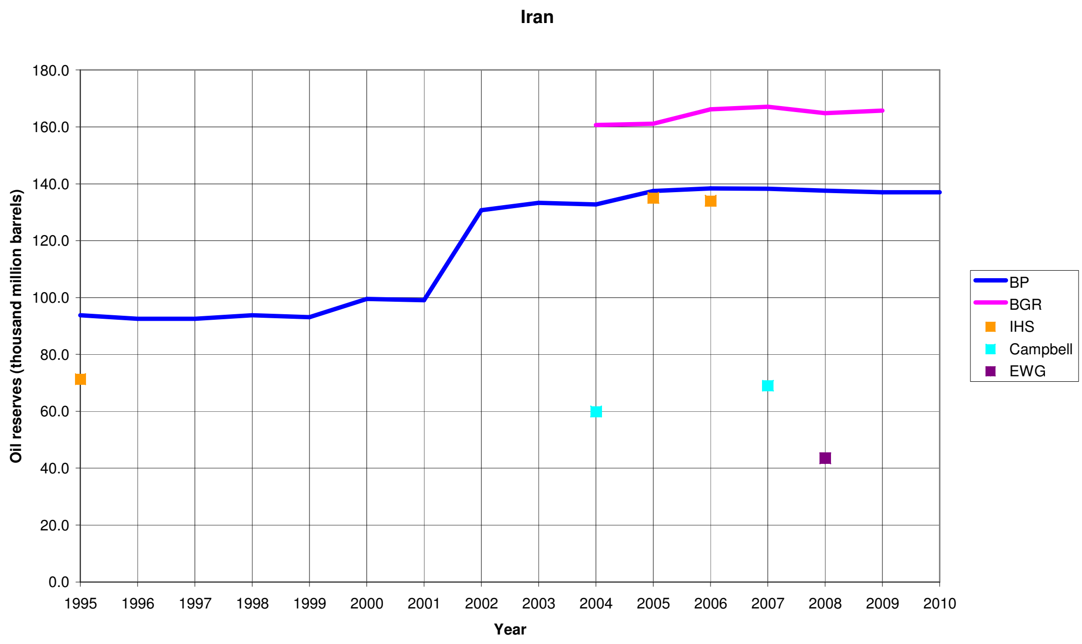

Fig 3.2: Comparison of Iranian oil reserves data from various sources

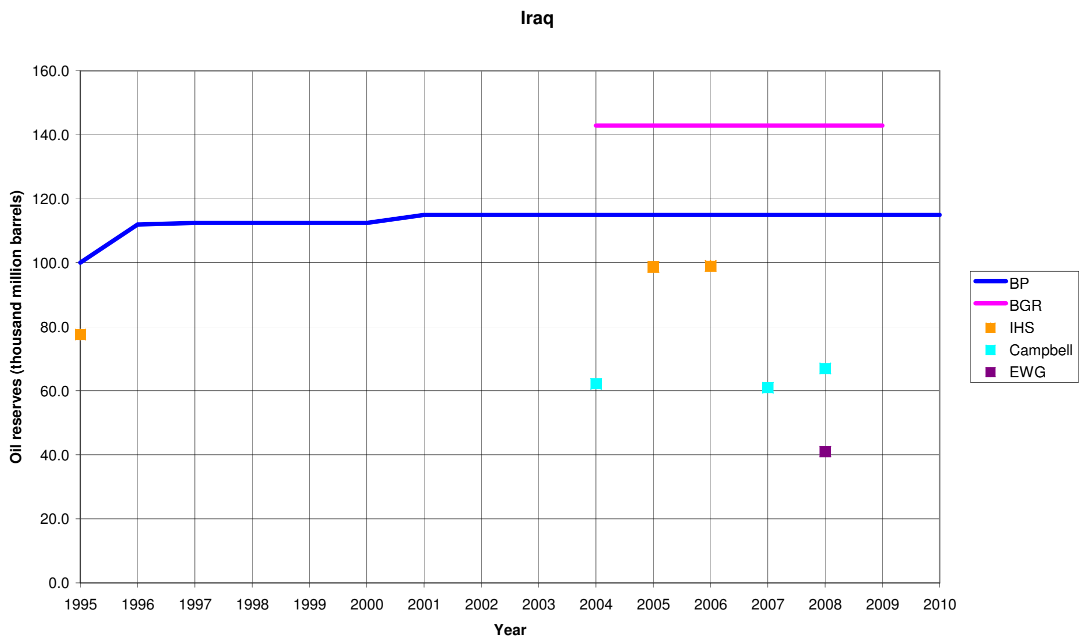

Fig 3.3: Comparison of Iraqi oil reserves data from various sources

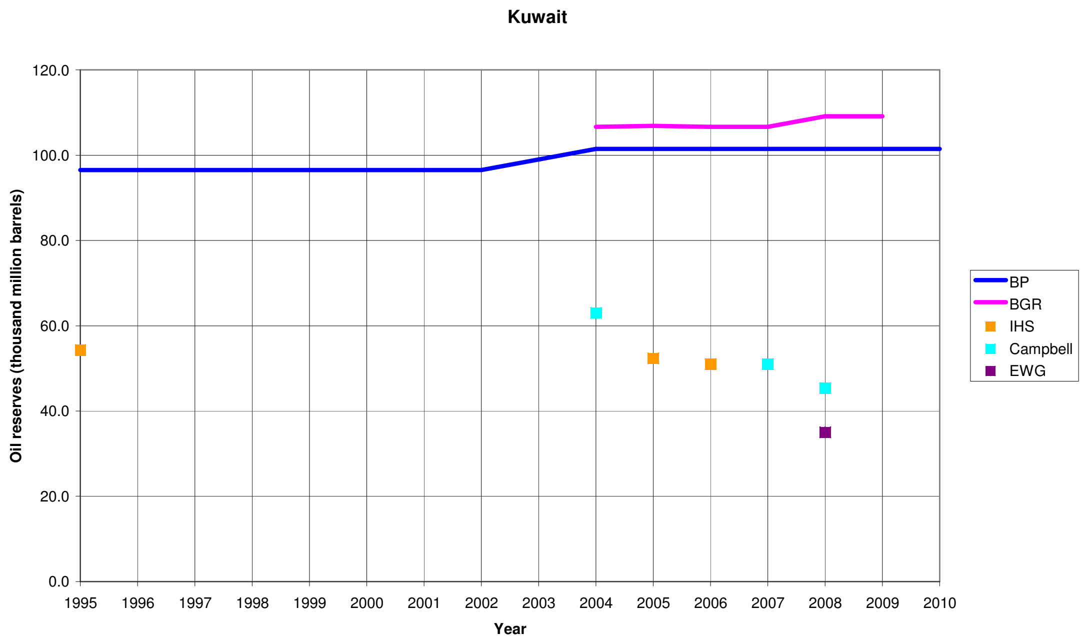

Fig 3.4: Comparison of Kuwaiti oil reserves data from various sources

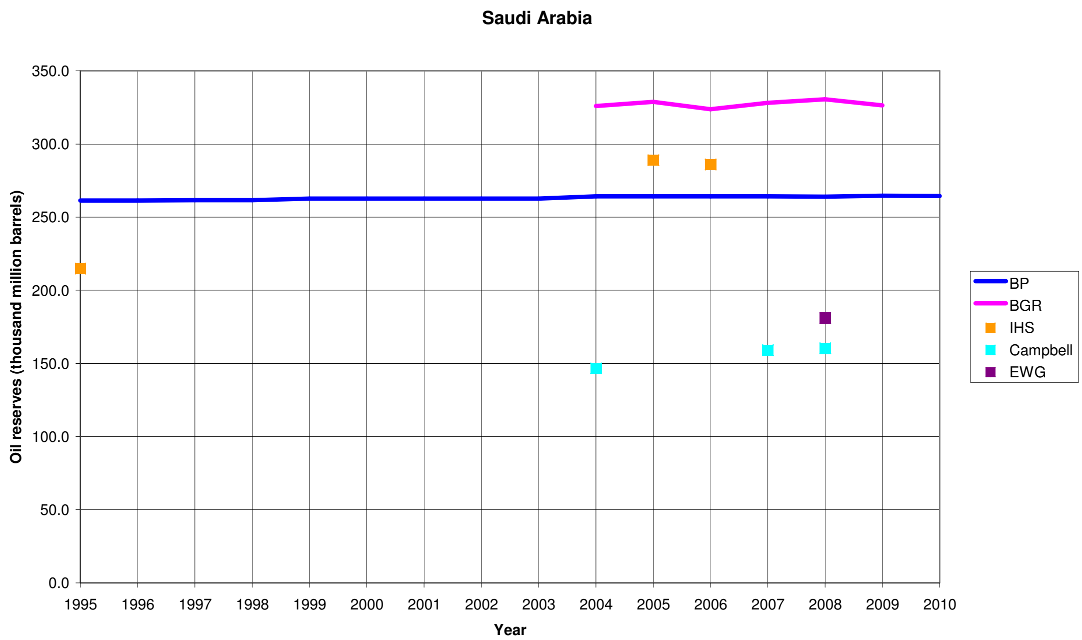

Fig 3.5: Comparison of Saudi Arabian oil reserves data from various sources

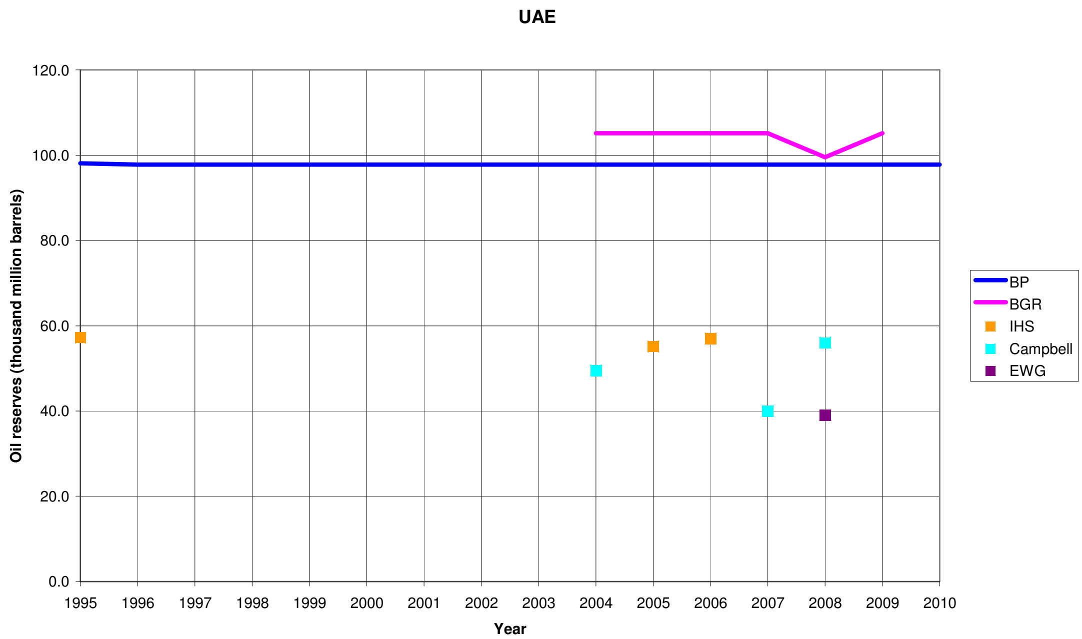

Fig 3.6: Comparison of United Arab Emirati oil reserves data from various sources

The BGR data seems to be a 2P estimate based on reported 1P figures. Indeed, McGlade (2010) suspects that BGR's estimates are mainly taken from publically available data sources as they match public 1P data. It is possible to use statistical estimation to derive a median (2P) value from a known P90 (1P) value. For example, Quirk and Ruthrauff (2006), who identify that the probability distribution takes a lognormal form and calculate – with their analysis of a variety of mature oil plays outside the Middle East – that the average value of the cumulative probability of the mean is 21%, could be used. However, this sort of estimation would not be useful if the 1P figures used were inaccurate as this report's analysis of their often flat, occasionally jumping and possibly overstated reserves suggests.

The remaining 2P figures are sourced from IHS, Campbell and EWG. The problem with these is that, with the exception of IHS figures of the 2000s for Saudi Arabia, they are all below the 1P figures. This is an issue because 1P figures should be less that 2P figures as they are more conservative by definition and also liable to underestimation resulting from incorrect aggregation (Sorrell and Speirs 2009). Apart from the 1P numbers being invented and having no basis in reality, the only other possible explanation for this difference could be more narrow definitions of oil. This may be true to some extent given the definitions used which were reviewed by McGlade (2010), and summarised here for the agencies concerned as follows:

Agency	Liquids included
BP	Conventional crude oil, extra heavy oil, condensate, shale oil, oil sands and NGL
BGR	Conventional crude oil >10°API, condensate and NGL
IHS (1995) / USGS	Conventional crude oil >17.5°API, NGL
IHS	Conventional crude oil, extra heavy oil, NGL, condensate, and oil sands
Campbell	Crude oil >17.5°API excluding shale oil, CTL, deepwater oil, polar oil, and NGL from gas plants Crude oil <15°API, shale oil, CTL, deepwater oil, polar oil, and NGL from gas plants
EWG	Conventional crude oil, condensate and NGL

Table 3.2: Liquids included in data by agency (source McGlade (2010))

However, considering the magnitude of the reserves discrepancies between the BP 1P data and the IHS, Campbell and EWG 2P data, and that most Middle Eastern oil is in the form of conventional crude anyway, the difference in definitions being the explanation seems unlikely.

On a global level, Strahan (2007) and Sorrell and Speirs (2009) both compare 1P reserves from BP with 2P reserves from IHS and identify that they are both approximately the same with the implication that one or both of the estimates are incorrect. The general conclusion reached by most authors is reflected by Bentley when he says that the proved reserves tell us nothing (Strahan 2007) and that reliance on them leads to misconceptions (Bentley 2007).

3.4. Sensitivity of reserves to the peak

Sorrell et al (2009) show that Hubbert style curve fitting models do not reflect reality and explain the reasons why they have been criticised; essentially that the original lognormal curve was a mathematical convenience and that they use a mechanistic approach paying insufficient attention to economic variables. Indeed, there is no physical explanation as why future production should follow any curve fitted to past production, and even if there was, any number of different curve forms could be fitted to such historical data. Sorrell et al (2009) go on to record the increasing replacement of these curve fitting models by disaggregate approaches. They propose a model where there are three factors which determine the peak, namely reserves, exponential growth rate and exponential decline rate. This report tests the sensitivity of the peak to reserves, so suitable growth rates and decline rates must now be determined.

3.4.1. Growth rate

The best estimate for this would be to look at historic production and determine a growth rate from that. Strahan (2007) suggests that OPEC production figures are widely considered unreliable, as they are another factor used to determine future production quotas, but not as much so as the reserves figures because they can be verified with much effort. Simmons (2005) suggests that even the OPEC Secretariat no longer trust the figures submitted by their members and now use third party reports to determine probable production. Consultancies that produce this sort of data include Petrologistics which counts tanker traffic and Groppe, Long & Littell which collates import data to make estimates. However, this data is not in the public domain and the BP Annual Statistical Review is again turned to. It helpfully provides production data figures going back to 1965 again in an electronically manipulable format. These figures are plotted for the countries concerned. An exponential regression was also performed on each country's data using spreadsheet software to give a curve of best fit which is also plotted on the graph (fig 3.7).

The problem with this approach of using exponential regression to derive a rate of growth from historic production data is that the rate will take into account periods of unusual production patterns. For example, the oil shocks of the 1970s and subsequent oil glut of the 1980s, and the Gulf War of 1990-91 where declines in Iraqi and Kuwaiti production were made up by Saudi Arabia. Possible remedies to this would be either to exclude such episodes from the data to be analysed or to only analyse data for the period after where any such unusual production patterns are known to have taken place. However neither solution was deemed to be satisfactory as the data point selection would be subjective. Therefore the full dataset was used and analysed on the understanding that the curves fitted pay attention to these unusual periods and therefore the rates derived may have a larger margin of error in them than might be expected.

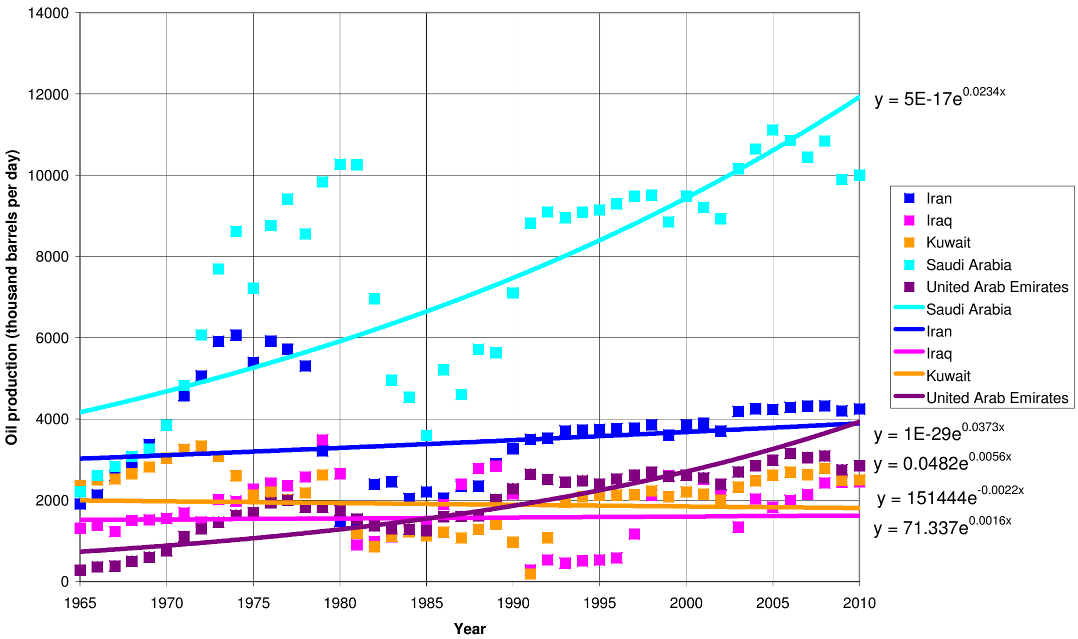

Fig 3.7: Historic oil production by country with exponential curves of best fit (source BP (2010))

The formulae given for the exponential regression follow the $x(t) = x_0 \bullet e^{kt}$ form. They can, however, be converted to the $x(t) = x_0 \bullet (1+r/100)^{t/p}$ form, which is more relevant when studying rates, using the formula:

$$k = \frac{\ln\left(1 + \frac{r}{100}\right)}{p}$$

The period p will be a fixed constant of 1 year so the formula can be substituted and rearranged to derive a rate r from the value of k:

$$r = 100\left(e^k - 1\right)$$

Using the k values from Fig 3.7, the growth rates can be worked out for each of the countries being examined:

	K	*r*
Iran	0.0056	0.6%
Iraq	0.0016	0.2%
Kuwait	-0.0022	-0.2%
Saudi Arabia	0.0234	2.4%
UAE	0.0373	3.8%

Table 3.3: Transformation of k to r

It is noted that k and hence r are negative for Kuwait. However, the magnitude is so negligible that it could be part of a plateau phase and thus the rate will be taken as 0%.

3.4.2. Decline Rate

For the model used in this study, the figure required is the production-weighted aggregate decline rate for all fields in the countries examined, including those in build up. The best analysis on decline rates has been done in IEA (2008) and they provided this figure both for OPEC at 3.3% and non-OPEC at 4.7% fields but omitted the global aggregate which was settled later by Sorrell et al (2009) as approximately 4.1%. Unfortunately country level figures are not available at least in the public domain, so the OPEC figure of 3.3%, which is more granular than the global figure, will be applied to the countries being examined as they are all OPEC members.

3.4.3. Results produced by model

The model proposed by Sorrell et al (2009) proposes that global production grows at an exponential rate to the peak and then declines at a fixed exponential rate from the peak. They explain that this will produce a sharp peak which is unlikely in practice, but serves as a simple approximation. Further, the model assumes that production will continue for 100 years following the peak by which time the cumulative production is essentially the URR.

In this study the model proposed by Sorrell et al (2009) is applied to each country being examined rather than to the whole world. The two numbers that will be applied for the growth rate and the decline rate of each country have been determined. A range of possible estimates for the remaining reserves of each has been identified in Figs 3.2 to 3.6, and the most recent years' production is known (given the caveats already discussed with regards to production data) and was plotted in Fig 3.7.

Spreadsheet software was used to create the model and produce results for the countries being examined. Two graphs were plotted for each country using the model, one giving production curves for the minimum and maximum estimates of remaining reserves, and the other plotting how the amount of remaining reserves will affect the year of peak. These are shown in Figs 3.8 to 3.17.

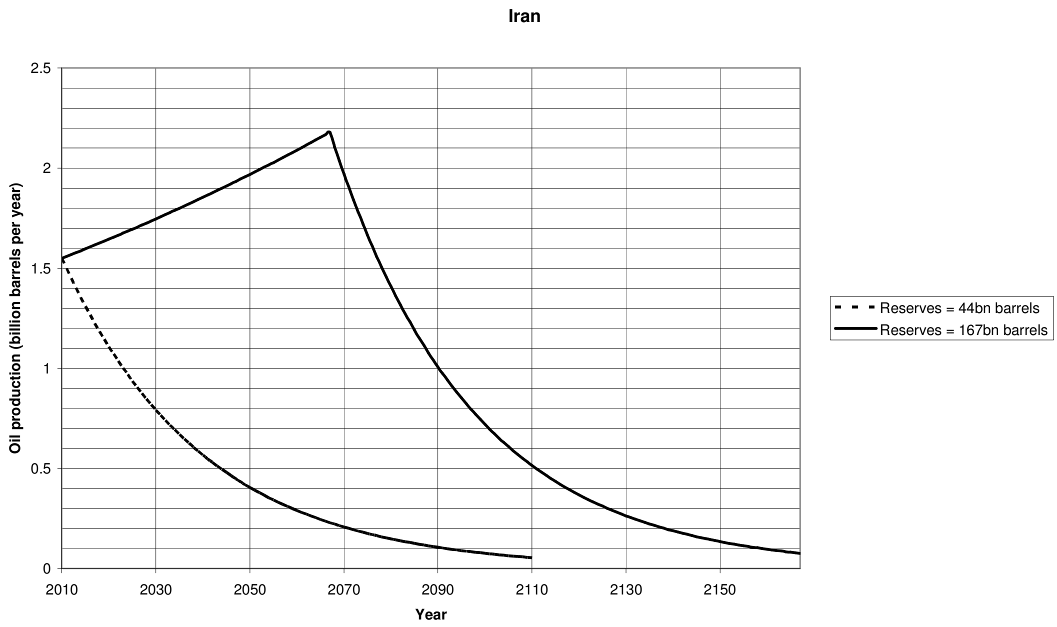

Fig 3.8: Projected Iranian production curves

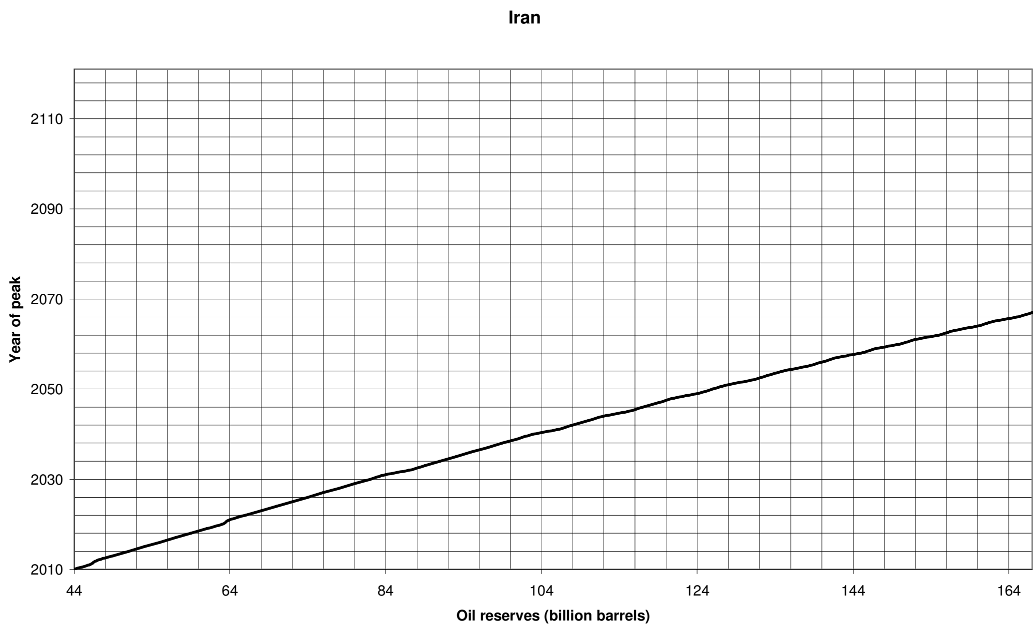

Fig 3.9: Variation of final Iranian peak production date with the size of reserves

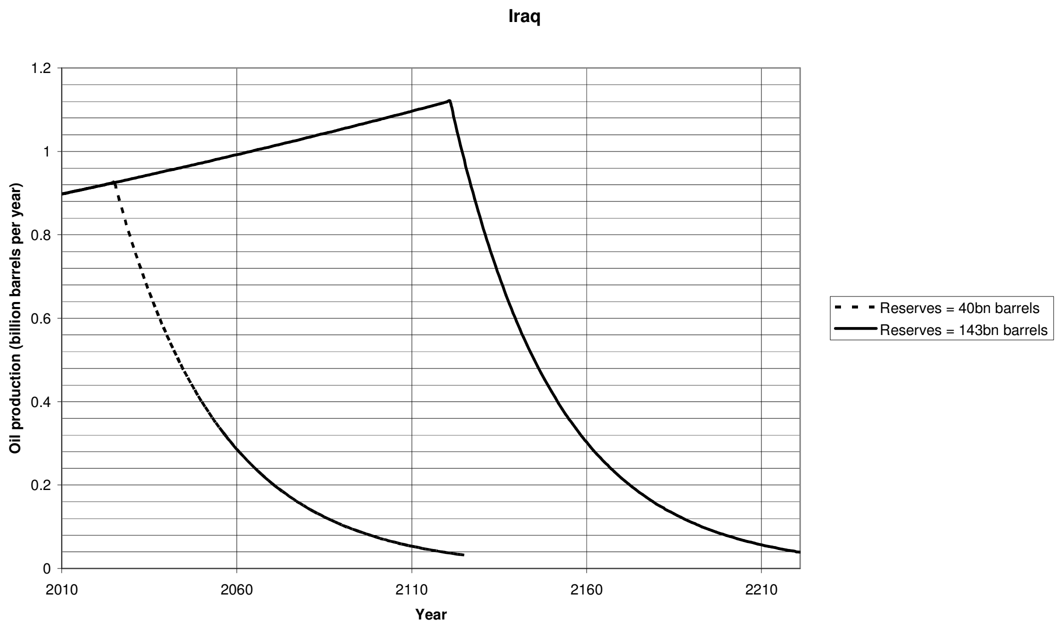

Fig 3.10: Projected Iraqi production curves

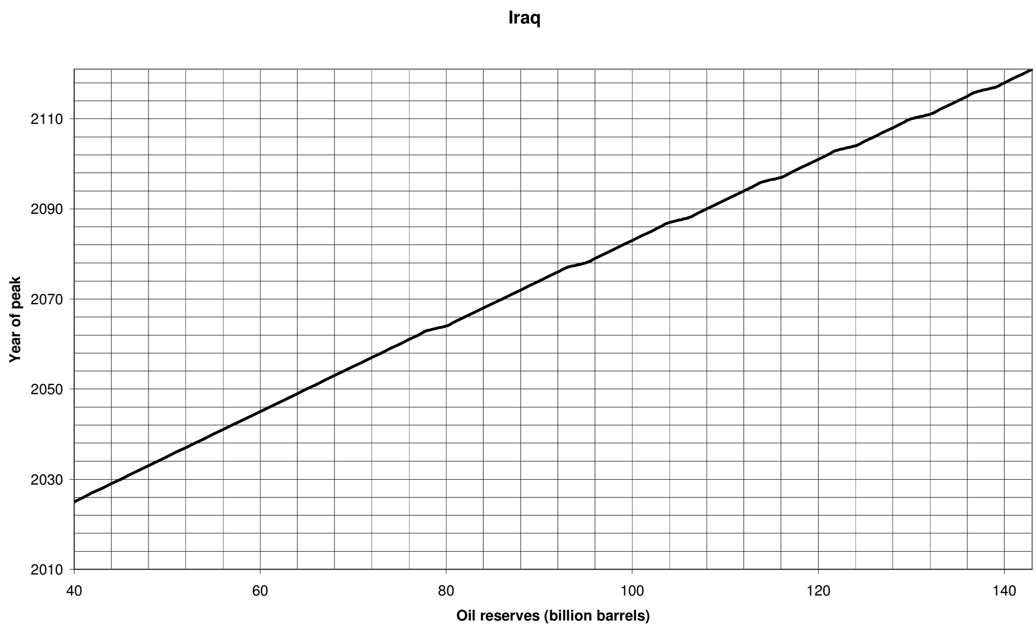

Fig 3.11: Variation of final Iraqi peak production date with the size of reserves

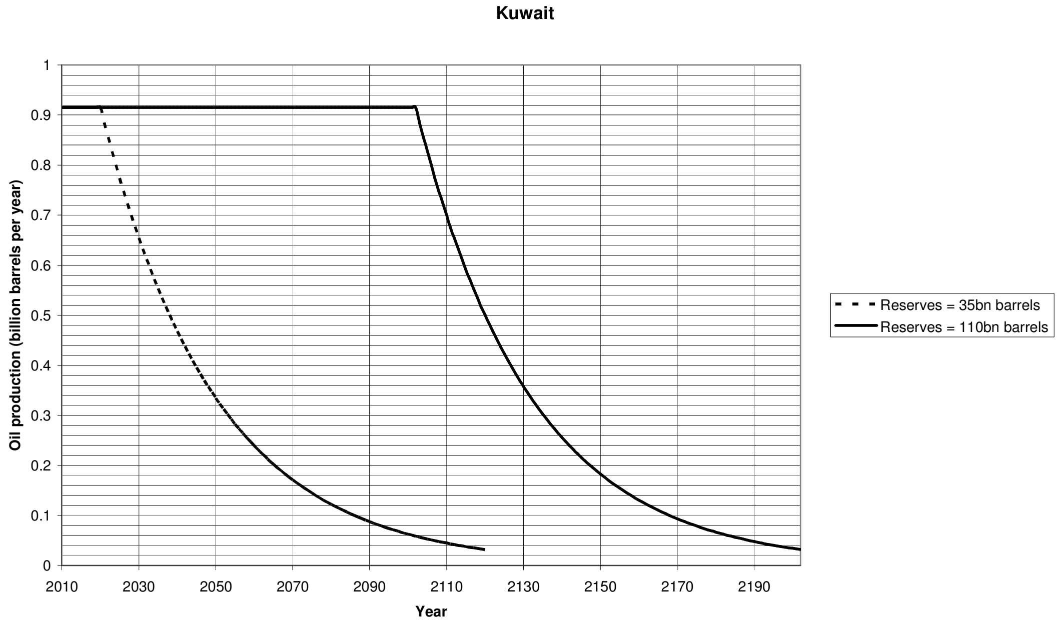

Fig 3.12: Projected Kuwaiti production curves

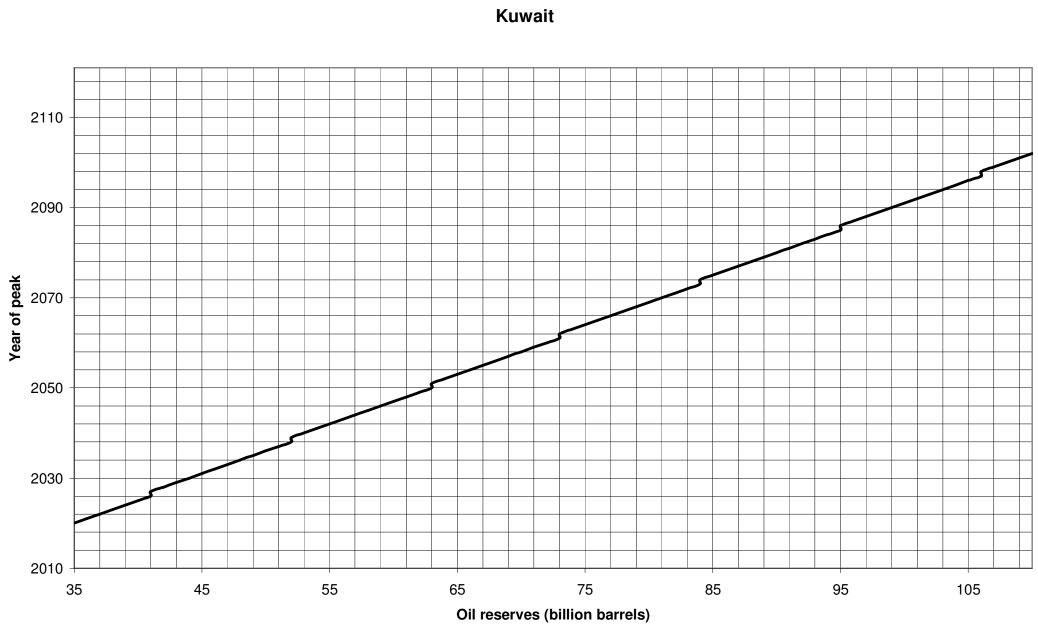

Fig 3.13: Variation of final Kuwaiti peak production date with the size of reserves

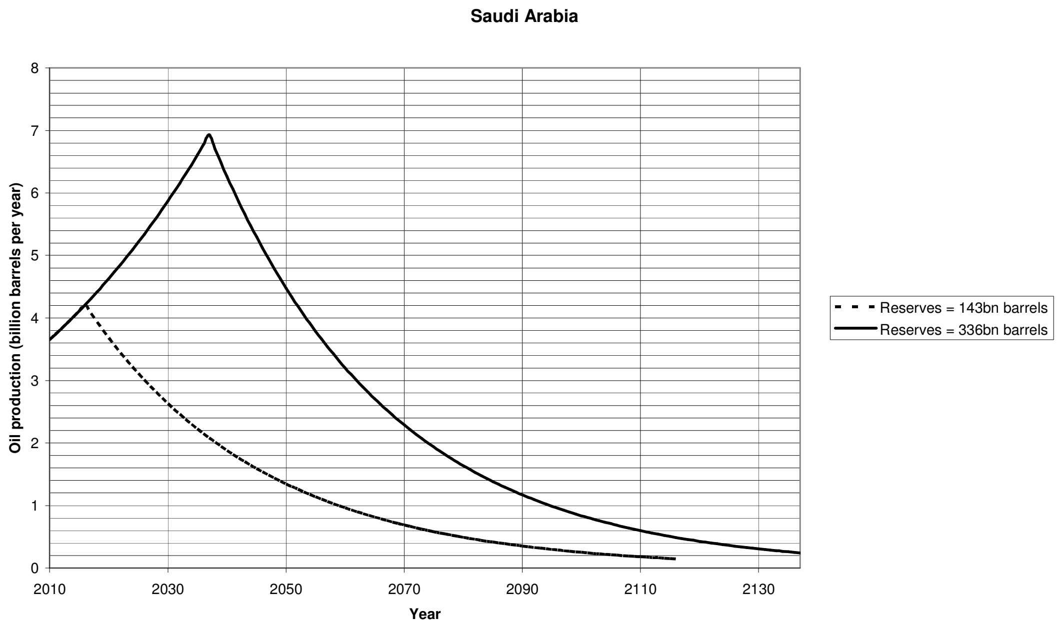

Fig 3.14: Projected Saudi Arabian production curves

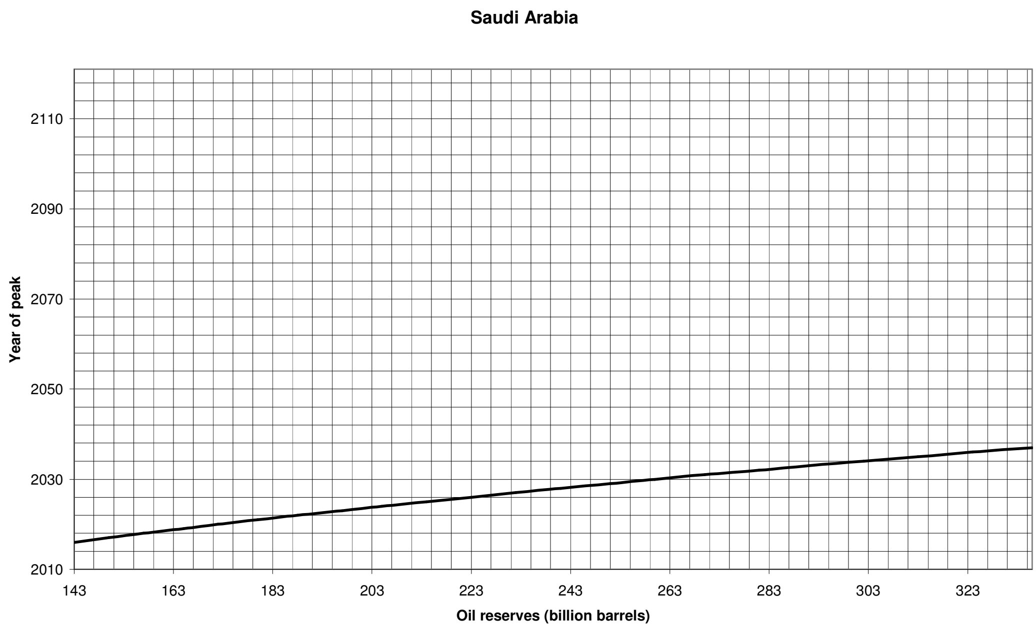

Fig 3.15: Variation of final Saudi Arabian peak production date with the size of reserves

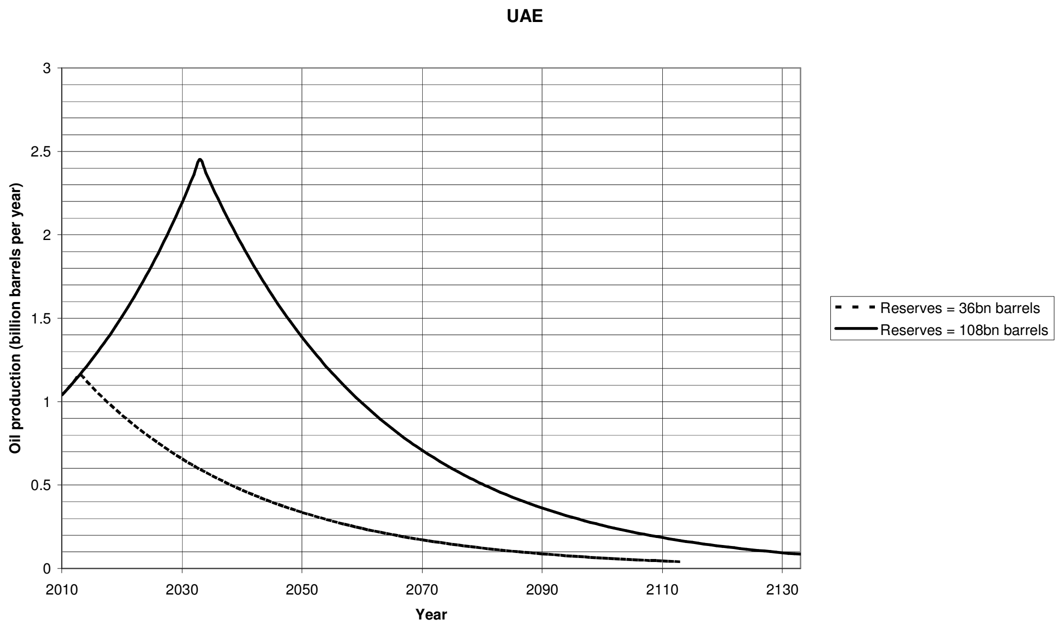

Fig 3.16: Projected United Arab Emirati production curves

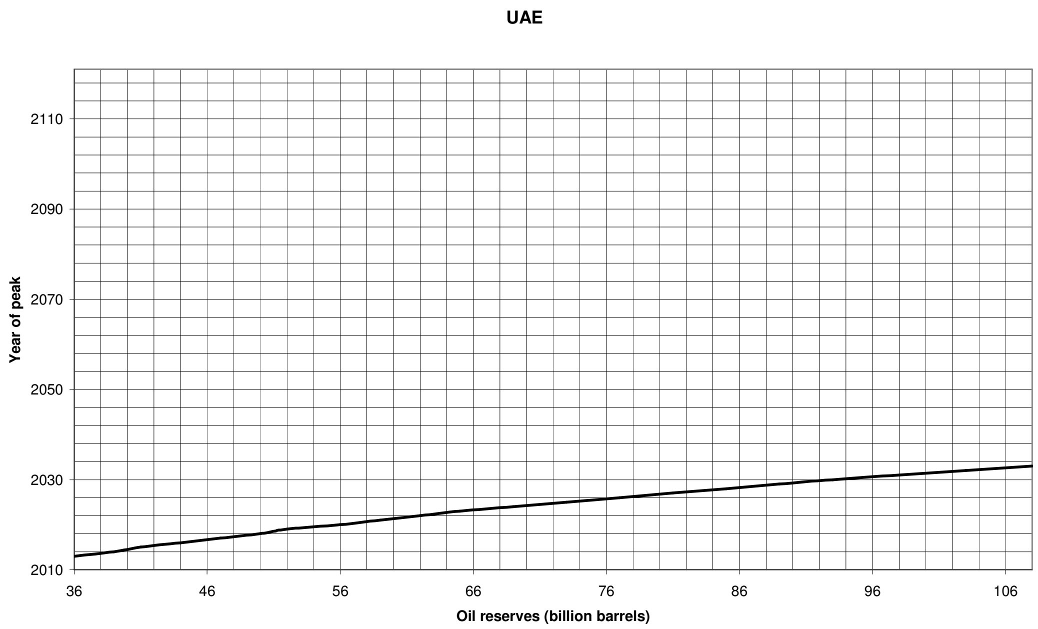

Fig 3.17: Variation of final United Arab Emirati peak production date with the size of reserves

3.4.4. Arithmetic interpretation of model

For any production curve, the area under the graph between any two dates will give the amount of oil extracted during that period. Therefore it is possible to use integration to express the model used as a mathematical formula relating remaining reserves to production and the date of the peak. The following formula was derived for the model used in this study with the workings given in the appendix:

$$\text{remainingReserves} = \int_{n+1}^{\text{peak}} \text{productionDuringGrowth}_n \bullet \left(1 + \tfrac{r_{\text{growth}}}{100}\right)^{t-n} dt + \int_{\text{peak}}^{\text{peak}+100} \text{productionDuringGrowth}_n \bullet \left(1 + \tfrac{r_{\text{growth}}}{100}\right)^{\text{peak}-n} \bullet \left(1 + \tfrac{r_{\text{decline}}}{100}\right)^{t-\text{peak}} dt$$

3.4.5. Aggregating the counties examined

The pessimistic and optimistic cases were summed for the countries examined into a single graph showing the range of likely dates when these OPEC states, as a group, will enter terminal decline:

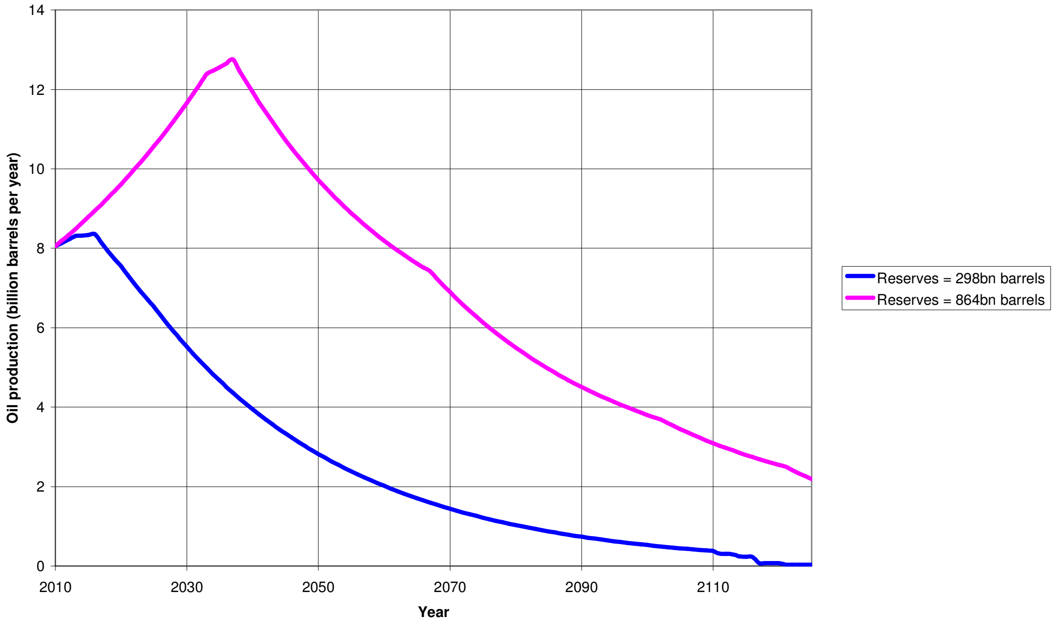

Fig 3.18: Projected aggregate production curves for the five OPEC countries with the largest reserves

4. Interpretation and discussion

As already outlined OPEC occupied the role of swing producer controlling a vast proportion of the worlds remaining conventional crude oil reserves. The state of those reserves is therefore of imperative importance to energy policy makers.

Figure 3.18 shows that the model used in this study predicts that the peak of OPEC production (the crude oil production of the OPEC states excluded from the study will not be enough to have any impact) could be as soon as 2016 on the basis of the most pessimistic assumptions of their reserves and as far away as 2037 on the most optimistic. As the majority of non-OPEC states are already at the peak or in decline (IEA 2008), it is certain that the global peak will occur before this OPEC peak. Indeed, it is interesting to note that the peaks predicted in aggregate for all the countries examined are exactly the same as the peaks predicted for Saudi Arabia which shows how much impact that single country has not only on OPEC production but also has on global production.

The model used is quite simplistic. Already mentioned is the unrealistic sharp peak produced at the intersection of the exponential growth and exponential decline curves. Exponential growth in itself may not be a good assumption. It does not fit the historic data particularly well (for reasons already explained), it would require a steadily growing investment each year in new production facilities and it is likely to result in a higher depletion rate (Section 2.2.2.). It is uncertain whether the countries concerned would be willing or able to make the investment required.

Some OPEC members are already wealthy; they may not need any extra income or have any incentive to grow production. For example, UAE's per capita income is among the highest in the world. It already provides all the social services that it citizens could ever possibly want: free education, housing and healthcare, and heavily subsidised energy. It is therefore no surprise that the Arab Spring has largely passed it by.

On the other hand, some other of the OPEC members examined may not have the cash available to make the investments required to sustain exponential rates of growth in oil production. For example, sanctions imposed against Iran by numerous nations and multinational entities have had a strong economic impact and the Iraq War is clearly a drag on production and investment there. Even with the extraordinarily high current oil prices Saudi Arabia is facing a fast shrinking surplus having to massively increase spending on welfare as a concession to stave of further political unrest.

Possible methods for releasing funds for social services might include privatising national oil companies or to opening up oil markets to outside firms, either of which would decrease the burden of governments to fund investment. Sales of national oil companies would raise money from the assets privatised and private companies would bring their own external capital. Individual countries are at varying places with regards to policy on this, Iraq, at one end, has already gone down the path since the Iraq War, whereas Iran, on the other, would have difficulties moving in the direction due to the sanctions it faces. It remains to be seen whether enough of an incentive will be realised in all the countries examined to go down this route.

Beyond that is the more fundamental question addressed by this study, which is whether the reserve levels will allow the major OPEC crude oil producers to continue delivering growth. Reserve levels have much uncertainty attached and there are already conflicting stories given on the remaining exploration potential of Saudi and Iraq. Simmons (2005), for instance, suggested that the state of Saudi Arabia's oil resources and production operations differed sharply from the then globally accepted Saudi version.

IEA (2008) give the actual crude oil production from OPEC for 2000 and 2007 (of which the aggregate figures from BP (2011) of the five countries examined in this study represent approximately 72%) and projected for 2015 and 2030 (in their Reference Scenario). Under their scenario and using the 72% ratio, their expectation of aggregate production of the five Middle Eastern OPEC countries examined in this report can be estimated and compared to the model used in this study:

Year	2000	2007	2015	2030
72% of IEA (2008) Reference Scenario for OPEC	21	22	26	28
BP (2011) for Iran, Iraq, Kuwait, Saudi Arabia and UAE	21	23		
Aggregate model with 298bn barrels of reserves			23	15
Aggregate model with 864bn barrels of reserves			24	32

Table 4.1: Comparison of production in millions of barrels of crude oil per day between IEA (2008) Reference Scenario and model produced for this study

This suggests that even under the IEA (2008) Reference Scenario – whose forecasts for crude oil production in 2030 are described by Aleklett et al (2009) as appearing significantly overstated – production will be less than under the most optimistic reserves assumptions of our model. This means that much more exploration and discovery would be required than even anticipated under the Reference Scenario, the chances of which seem very unlikely. All this leads to the conclusion that the most optimistic view of reserves used by this study is likely infeasible. Even to get production up to the levels required for the most pessimistic forecast of peak in 2016, the investment has to have been committed already. This may indeed be the case for Saudi, Kuwait and UAE. But whatever the case, investment decisions need to be made at least a decade before, when the money must already be lined up.

The global oil price isn't set by OPEC directly, but by the cost of the most marginal, expensive to extract barrel, which is quite probably produced somewhere in the US. Consequently, in a post-peak world, OPEC will have little price control, and prices will rise regardless of whether demand increases as expected. One possibility is a future of saw-tooth economic conditions, where oil prices rise, break economies and collapse back. Companies or states may then not want to risk investing over the 10-20 year time frame of developing an oil field, from starting exploration to producing enough oil to break even. Under this scenario OPEC may continue producing its easily and cheaply extractable crude oil at little marginal cost, but with the decline in supply from lack of new projects being pursued the global price will still rocket when there isn't enough to meet demand. Indeed, it is likely that price stress will occur even before peak oil, because (if the global economy recovers from its current lull) demand is likely to rise faster than supply and outstrip it, even if new discoveries are being made and put into production. This could lead to an economic signal of price rises before the peak is even reached.

In summary, using even the most optimistic reserves assumption, the model used suggests that global crude oil supply will go into decline from 2037, and anything less makes things tight even earlier. Given the lead times of major oil projects, the problems of peak oil are probably already with us, even if the early symptoms of rising oil prices seem bearable.

5. Conclusions

5.1. Summary conclusions of the study

- Geology, technology, price and energy return on investment are the factors which will limit the upward revision of crude oil reserves and will also control the production profiles that are achieved in a field or basin.
- Reserves uncertainty comes from many sources including: differing definitions of crude oil; imprecise alignment and use of reserves classification systems and methods; incorrect methods of aggregation (arithmetic rather than probabilistic addition is often used); and inaccurate reporting by national governments (perhaps for political or economic gain) which can be observed historically from flat reserves reporting, reserves jumps by OPEC members during the 1980s and overstatement of reserves by Mexico that were owned up to in 1997.
- The study focuses on the five Middle Eastern OPEC countries with the largest reserves as they account for a vast amount of the remaining world crude oil reserves and swing producer status while at the same time having little transparency or openness, making it difficult for energy policy decisions to be made.
- The model predicts that given reasonable assumptions made about the rate of supply growth before, and decline rate after the peak, in aggregate the five countries will peak between 2016 and 2037 depending on the reserves estimate. This suggests that the OPEC peak and indeed the global peak will be within this timeframe.

- When analysing against the WEO(2008) model and the critique of that model by Aleklett et al (2009), the upper end of the range of dates for the peak appears to be over optimistic and even to reach the middle of the range would require massive investment today due to industry lead times.
- It is therefore probable that the high oil prices being experienced today are the early symptoms of the peak oil problem appearing.

5.2. Future directions of research

The model used, being the cornerstone of this study, would perhaps be the focus for future research:

- Better estimates for the inputs that are entered into the model could be used. For example, the production growth rate was estimated using exponential regression on historic production but the curve had a large margin of error for reasons that have been outlined, and the depletion rate used was an OPEC level figure whereas an individual country level figure (perhaps estimated using reserves depletion rate equivalence although data availability in the public domain is very limited) may produce better results as it would use more granular estimates.
- The model could be applied to more countries perhaps to give a better prediction of when the OPEC or global peak will be.
- The model's predictions could be back tested against countries already known to be in decline, to see how well it works when compared to what actually happened.

- The complexity of the model could be increased as it is currently quite simplistic. It could be developed further to better fit the historic data, to produce a more realistic peak or plateau, and/or to include some economic input. It would be interesting to extend the model to include the relationship between global economic activity, oil demand and oil price. The saw-tooth forecast of economies/oil price/oil supply could be what happens but a more rigorous analysis would be useful.
- Non-conventional sources of oil were excluded from the report as it was felt that they would have added too much complexity to the study, but as an additional piece of research they could be considered and the model could perhaps be applied to them. Also, a review of whether and from which sources a replacement of oil, as an easily transportable and highly energy dense source of energy, might come from would be a useful additional piece of research considering how close the model predicts the peak to be.

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7. Appendix

7.1. Letter to OPEC

Clockwork Underwriting LLP
215-217 Victoria Park Road LONDON E9 7HD
www.clockworkgroup.org

12 May 2011

Public Relations and Information Department
OPEC Secretariat
Obere Donaustrasse 93
A-1020 Vienna
Austria

Dear Sirs

I am conducting some academic research as a part time post graduate at the Aon Benfield Hazard Research Centre, University College London (UCL).

I am looking for a reliable description of how OPEC quotas are set between its member countries. I have not been able to find this from public sources so I was wondering if you would be able to help.

I am particularly interested to find out:

- If there is there a formula that is used to calculate each member country's quota?
- Whether the criteria for quota allocations to member countries has changed over time since OPEC's formation and how?
- What factors affect a member country's quota e.g. current production levels, size of reserves etc.?

I would be most grateful if you could let me have any information you might be able to provide in this area.

Thanking you

Yours faithfully

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Fig 7.1: Letter sent to OPEC Secretariat to which no reply was received

7.2. Workings deriving mathematical expression of the model used

An exponential production curve can be expressed in the form:

$\text{production}(t) = \text{production}_0 \bullet (1+r/100)^t$

The production curve of the model used has two components, the growth phase from the year following current production (at year *n*) to the peak and the decline phase for 100 years following the peak:

$$\text{remainingReserves} = \int_{n+1}^{\text{peak}} \text{productionDuringGrowth}(t)dt + \int_{\text{peak}}^{\text{peak}+100} \text{productionDuringDecine}(t)dt$$

7.2.1. Growth phase

$\text{productionDuringGrowth}(t) = \text{productionDuringGrowth}_0 \bullet (1+r_{\text{growth}}/100)^t$

The unknown in the growth phase is $\text{productionDuringGrowth}_0$. However, production in the last year for which data is available, *n*, is known. Therefore the formula can be rearranged to express $\text{productionDuringGrowth}_0$ in terms of year *n* and productionDuringGrowth for that year:

$$\text{productionDuringGrowth}_0 = \frac{\text{productionDuringGrowth}_n}{\left(1+\frac{r_{growth}}{100}\right)^n}$$

This can be substituted into the original formula to give a complete growth phase curve formula:

$$\text{productionDuringGrowth}(t) = \frac{\text{productionDuringGrowth}_n}{\left(1+\frac{r_{\text{growth}}}{100}\right)^n} \bullet \left(1+\frac{r_{\text{growth}}}{100}\right)^t$$

Simplifying gives:

$$\text{productionDuringGrowth}(t) = \text{productionDuringGrowth}_n \bullet \left(1+\frac{r_{\text{growth}}}{100}\right)^{t-n}$$

7.2.2. Decline phase

$$\text{productionDuringDecline}(t) = \text{productionDuringDecline}_0 \bullet (1+r_{\text{decline}}/100)^t$$

For the Decline phase, again the unknown is production at year 0. However we know that the decline phase curve crosses the growth phase curve at the year of the peak, so:

$$\text{ProductionDuringDecline}_{\text{peak}}=\text{productionDuringGrowth}_{\text{peak}}$$

Substituting gives:

$$\text{productionDuringDecline}_0 \bullet \left(1+\frac{r_{\text{decline}}}{100}\right)^{\text{peak}} = \text{productionDuringGrowth}_n \bullet \left(1+\frac{r_{\text{growth}}}{100}\right)^{\text{peak}-n}$$

Rearranging gives:

$$\text{productionDuringDecline}_0 = \frac{\text{productionDuringGrowth}_n \bullet \left(1 + \frac{r_{\text{growth}}}{100}\right)^{\text{peak}-n}}{\left(1 + \frac{r_{\text{decline}}}{100}\right)^{\text{peak}}}$$

Substituting back into original formula gives:

$$\text{productionDuringDecline}(t) = \frac{\text{productionDuringGrowth}_n \bullet \left(1 + \frac{r_{\text{growth}}}{100}\right)^{\text{peak}-n}}{\left(1 + \frac{r_{\text{decline}}}{100}\right)^{\text{peak}}} \bullet \left(1 + \frac{r_{\text{decline}}}{100}\right)^{t}$$

Simplifying gives:

$$\text{productionDuringDecline}(t) = \text{productionDuringGrowth}_n \bullet \left(1 + \frac{r_{\text{growth}}}{100}\right)^{\text{peak}-n} \bullet \left(1 + \frac{r_{\text{decline}}}{100}\right)^{t-\text{peak}}$$

7.2.3. Integration

Substituting final formulas for growth and decline phases into the formula for reserves gives the model:

$$\text{remainingReserves} = \int_{n+1}^{\text{peak}} \text{productionDuringGrowth}_n \bullet \left(1 + \frac{r_{\text{growth}}}{100}\right)^{t-n} dt + \int_{\text{peak}}^{\text{peak}+100} \text{productionDuringGrowth}_n \bullet \left(1 + \frac{r_{\text{growth}}}{100}\right)^{\text{peak}-n} \bullet \left(1 + \frac{r_{\text{decline}}}{100}\right)^{t-\text{peak}} dt$$